# CLIC spoilers : thermal study

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# Outline

1. Introduction
2. Mathematical basis
3. Ionisation process
4. Image current heating process
5. Application to CLIC
6. Conclusion

Introduction

## AT CLIC:

- No massive absorber can survive the impact of even a single bunch train
  $\Rightarrow$ Use of a thin spoiler (length $\approx$ half a radiation length) to increase the divergence of the beam, and installation of an absorber somewhere downstream
- Aim of this study : specify the beam sizes for a given material to avoid damages

Mathematical basis

## Resolution of a linear differential equation via an integral method

$$D(f) = \sum_{i_1,i_2,i_3} a_{i_1,i_2,i_3}(x,y,t)\,\frac{\partial^{i_1+i_2+i_3} f}{\partial x^{i_1}\,\partial y^{i_2}\,\partial t^{i_3}} = 0 \qquad (E)$$

$$\begin{aligned}(x,y,t) \quad &\longrightarrow \quad G_{x_0,y_0}(x,y,t) \text{ solution of } E\\ U\times V\times\mathbb{R}^+ \quad &\longrightarrow \quad \mathbb{R}\\ \text{with } G_{x_0,y_0}(x,y,0) \quad &= \quad \delta(x-x_0, y-y_0)\ \ \forall\,(x_0,y_0)\in U\times V\end{aligned}$$

then for any function $f(x,y)$ with $(x,y)\in U\times V$

$$F(x,y,t) = \int_U\int_V f(x_0,y_0)G_{x_0,y_0}(x,y,t)dy_0dx_0$$

is solution of $E$ with $F(x,y,t=0) = f(x,y)$.

Mathematical basis

# Solution of the heat equation for Dirac initial condition

$$\frac{\partial T}{\partial t} = D\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \text{ with } D = K/C_v$$

- Infinite medium

$$\widetilde{T}_{x_0,y_0}(x,y,t) = \frac{1}{4\pi Dt}\exp\left(\frac{-(x-x_0)^2-(y-y_0)^2}{4Dt}\right)$$

- semi-infinite medium $y \geq 0$ : boundary condition $\frac{\partial \widetilde{T}_{x_0,y_0\geq 0}(x,0,t)}{\partial y} = 0$

$$\widetilde{T}_{x_0,y_0\geq 0}(x,y,t) = \widetilde{T}_{x_0,y_0}(x,y,t) + \widetilde{T}_{x_0,y_0}(x,-y,t)$$

$$\Leftrightarrow \widetilde{T}_{x_0,y_0\geq 0}(x,y,t) = \frac{\exp\left(-\frac{(x-x_0)^2}{4Dt} - \frac{y^2+y_0^2}{4Dt}\right)\cosh\left(\frac{y_0 y}{2Dt}\right)}{2\pi Dt}$$

Mathematical basis

## Solution of the heat equation for any initial conditions $f(x, y)$

A convolution integral to compute

- infinite medium

$$\iint_{\mathbb{R}^2} \frac{1}{4\pi Dt} \exp\left(\frac{-(x-x_0)^2-(y-y_0)^2}{4Dt}\right) f(x_0, y_0)dx_0dy_0$$

- semi-infinite medium

$$\int_{\mathbb{R}}\int_{\mathbb{R}+} \frac{\exp\left(-\frac{(x-x_0)^2}{4Dt}-\frac{y^2+y_0^2}{4Dt}\right)\cosh\left(\frac{y_0 y}{2Dt}\right)}{2\pi Dt} f(x_0, y_0)dx_0dy_0$$

Ionisation process

# Single bunch heating

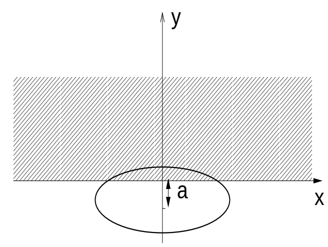



Figure 1: beam running into the collimator and centered in $y = a$

Hypothesis of work :

- Gaussian bunch
- Energy deposit uniform in the $s$ direction
- Diffusion negligeable over the duration of a bunch $\sigma_t = c/\sigma_z$

$$\Rightarrow \Delta T_{ion}(x, y, t = 0) = \frac{dE/dz \times N_p}{2\pi C_v \sigma_x \sigma_y} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{(y-a)^2}{2\sigma_y^2}\right)$$

Ionisation process

# Multiple bunch heating

- Diffusion of one bunch : calculation of the convolution integral (1D for the semi-infinite medium in $Y \times$ 1D for the infinite medium in $X$)

$$\Delta T_{ion}(x,y,t) = \frac{dE/dz \times N_p}{2\pi C_v \sqrt{(2Dt+\sigma_x^2)}} \exp\left(-\frac{x^2}{2}\frac{1}{2Dt+\sigma_x^2}\right) \times f(y,t)$$

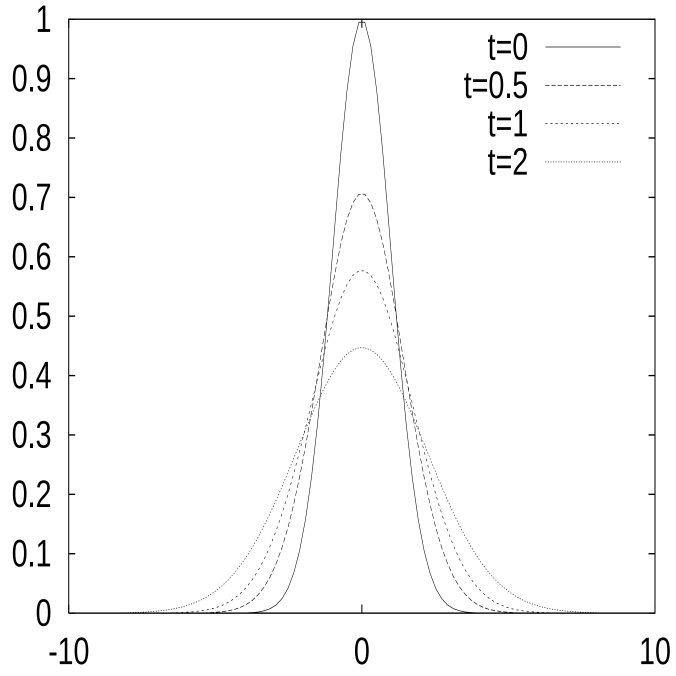
$$\text{with } f(y,t) = \frac{\sigma_y}{2\sqrt{2Dt+\sigma_y^2}} \exp\left(-\frac{(y-a)^2}{2(2Dt+\sigma_y^2)}\right)\left(1+\text{erf}\left(\frac{2aDt+y\sigma_y^2}{2\sigma_y\sqrt{Dt(2Dt+\sigma_y^2)}}\right)\right)$$

$$+\frac{\sigma_y}{2\sqrt{2Dt+\sigma_y^2}} \exp\left(-\frac{(y+a)^2}{2(2Dt+\sigma_y^2)}\right)\left(1+\text{erf}\left(\frac{2aDt-y\sigma_y^2}{2\sigma_y\sqrt{Dt(2Dt+\sigma_y^2)}}\right)\right)$$

- Solution for a bunch train : $N$ bunches, bunch spacing $\delta_t$ ($t > (N-1)\delta_t$)

$$\Rightarrow \Delta T_{ion}(x,y,t) = \frac{dE/dz \times N_p}{2\pi C_v} \sum_{n=0}^{N-1} \frac{e^{-\frac{x^2}{2}\frac{1}{2D(t-n\delta_t)+\sigma_x^2}}}{\sqrt{(2D(t-n\delta_t)+\sigma_x^2)}} \times f(y, t-n\delta_t)$$

Ionisation process : multiple bunch heating



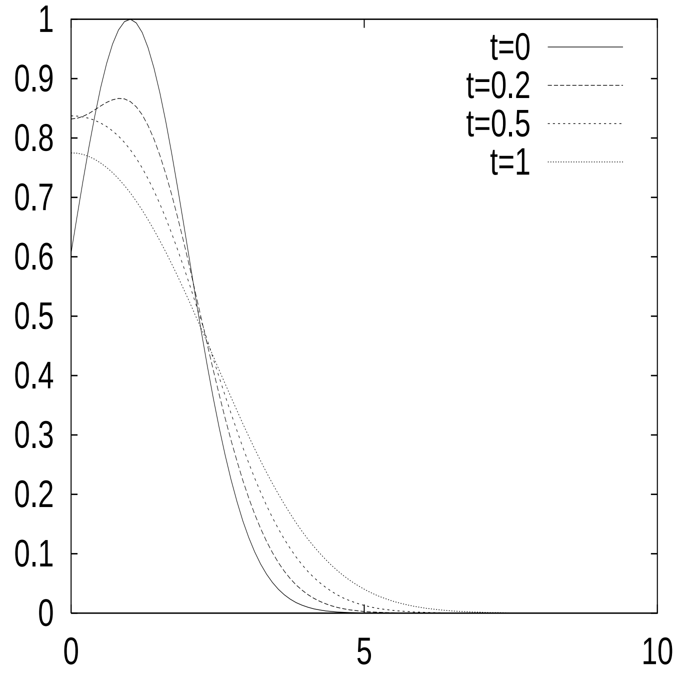



Image current heating process

## Single bunch heating

Hypothesis of work :

- same ones than for ionisation
- non magnetic metal + flat spoiler surface
- frequency under 100THz and $\sigma_x >> \delta = \sqrt{\frac{2\omega_z}{\mu_0 \sigma}}$ skin depth at $\omega_z = \sigma_z / c$

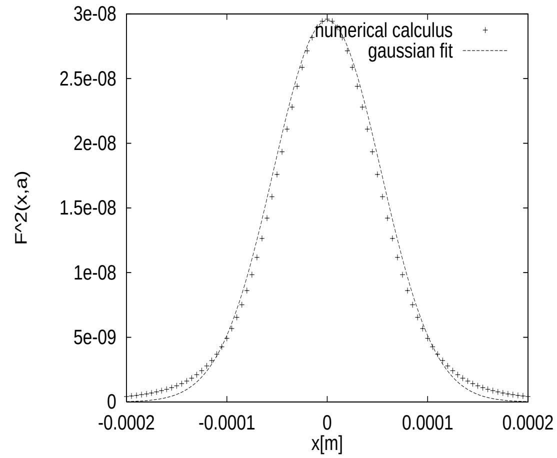
$$\Rightarrow \Delta T_{im}(x, y, t = 0) = \frac{Z_0 c}{2\pi C_v} \frac{Q^2}{\pi^2 \sigma_z^2} \frac{F^2(x, a, t = 0)}{4\pi^2 \sigma_x^2 \sigma_y^2} g(y/\delta, t = 0)$$

Xiantian E. Lin and David H. Whittum. *Image current heating on metal surface due to charged bunches.* Physical review Special Topics, Accelerators and beams,Vol 3, 101001(2000).

Image current heating process, Single bunch heating

$$\text{where } F(x,a,t=0) = \int_{-\infty}^{0}\int_{-\infty}^{+\infty} \frac{y_0 e^{-x_0^2/2\sigma_x^2} e^{-(y_0-a)^2/2\sigma_y^2}}{y_0^2 + (x_0-x)^2} dx_0 dy_0$$

which is approximated by a gaussian




$$F^2(x,a,t=0) = F^2(0,a)e^{-x^2/2\sigma_{gx}^2} \Rightarrow \sigma_{gx} = \frac{1}{\sqrt{2\pi}F^2(0,a)} \int_{\mathbb{R}} F^2(x,a,t=0)dx$$

Image current heating process, Single bunch heating

$$\text{where } g(y/\delta, t=0) = \int_0^{+\infty} e^{-z} e^{-2z^{1/4} y/\delta} dz$$

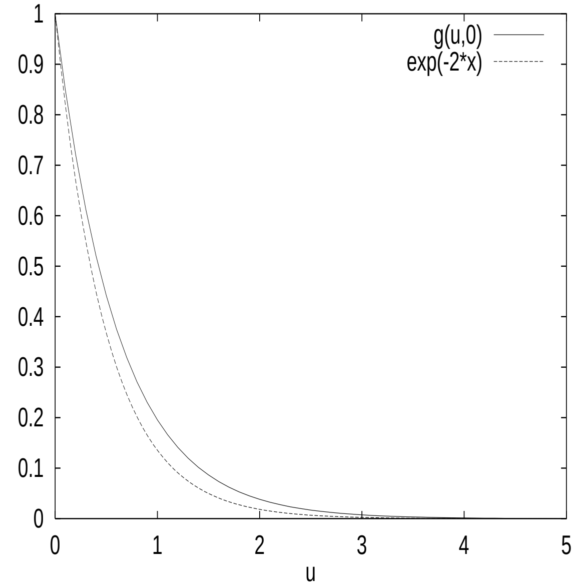



Figure 2: $u \to g(u, t=0)$ $(u = y/\delta)$

Image current heating process

## Multiple bunch heating

- diffusion of the x-dependant part = diffusion of a gaussian in an infinite medium

$$\Rightarrow F^2(x,a,t) = \frac{F^2(0,a)\sigma_{gx}}{\sqrt{2Dt+\sigma_{gx}^2}} \exp\left(-\frac{x^2}{2}\frac{1}{2Dt+\sigma_{gx}^2}\right)$$

- diffusion of y-dependant part = diffusion of $g(y/\delta, t=0)$ in a semi-infinite medium
- finally for N bunches ($t > (N-1)\delta_t$):

$$\Delta T_{im}(x,y,t) = \frac{Z_0 c}{2\pi C_v}\frac{Q^2}{\pi^2\sigma_z^2}\frac{F^2(0,a)}{4\pi^2\sigma_x^2\sigma_y^2}\sigma_{gx} \sum_{n=0}^{N-1}\frac{\exp\left(-\frac{x^2}{2}\frac{1}{2D(t-n\delta_t)+\sigma_{gx}^2}\right)}{\sqrt{2D(t-n\delta_t)+\sigma_{gx}^2}} \times g(y/\delta, t-n\delta_t)$$

Image current heating process, Multiple bunch heating

$$g(y/\delta, t) = \int_{\mathbb{R}+} \frac{\exp\left(-\frac{y^2+y_0^2}{4Dt}\right)\cosh\left(\frac{y_0 y}{2Dt}\right)}{\sqrt{\pi Dt}} g(y_0/\delta)dy_0$$

$$\Leftrightarrow \quad g(y/\delta, t) = \frac{1}{\sqrt{\pi Dt}} \int_{\mathbb{R}+} \exp\left(-\frac{y^2+y_0^2}{4Dt}\right) \cosh\left(\frac{y_0 y}{2Dt}\right) \int_0^{+\infty} e^{-z} e^{-2z^{1/4} y_0/\delta} dz$$

$$\Leftrightarrow \quad g(y/\delta, t) = \frac{e^{-y^2/4Dt}}{2} \times$$

$$\int_0^{+\infty} e^{-z} \exp\left(\left(\frac{y}{2\sqrt{Dt}} - \frac{2z^{1/4}\sqrt{Dt}}{\delta}\right)^2\right) \operatorname{erfc}\left(\frac{2z^{1/4}\sqrt{Dt}}{\delta} - \frac{y}{2\sqrt{Dt}}\right) dz$$

$$+ \frac{e^{-y^2/4Dt}}{2} \times$$

$$\int_0^{+\infty} e^{-z} \exp\left(\left(\frac{y}{2\sqrt{Dt}} + \frac{2z^{1/4}\sqrt{Dt}}{\delta}\right)^2\right) \operatorname{erfc}\left(\frac{2z^{1/4}\sqrt{Dt}}{\delta} + \frac{y}{2\sqrt{Dt}}\right) dz$$

Application to CLIC

## Data : fatigue limits

$$T_{lim} = \frac{T^K_{melt}}{2} - T^K_{room} \text{ (C. Hauviller)}$$

Table 1: limit temperature variation above room temperature for fatigue

| | Be | C | Al | Ti | Cu | W |
|---|---|---|---|---|---|---|
| limit temperature variation $\Delta T_l\,[K]$ | 482 | 1618 | 173 | 673 | 385 | 1548 |

## Specific heat

$$C_v = 9R\left(\frac{T}{\theta_d}\right)\int_0^{\theta_d/T}\frac{x^4e^x}{(e^x-1)^2}\,dx \text{ (Debye)}$$

we chose $C_v = \frac{C_v(\text{room})+C_v(\text{fatigue})}{2}$

for beryllium : $C_v(\text{room}) = 1.94$ Jcm$^{-3}$K$^{-1}$ and $C_v(\text{fatigue}) = 4.12$ Jcm$^{-3}$K$^{-1}$

Application to CLIC, Data

## Thermal conductivity $K$ and electrical one $\sigma$

- We found $\sigma(T)$ in tables (J. Pett) and we chose :

$$\sigma = \frac{\sigma(\text{room}) + \sigma(\text{fatigue})}{2}$$

for copper : $\sigma(\text{room}) = 60\,10^6\ \Omega^{-1}\text{m}^{-1}$ and $\sigma(\text{fatigue}) = 23.4\,10^6\ \Omega^{-1}\text{m}^{-1}$

- *Wiedemann-Franz* law :

$$\frac{K}{\sigma} = \frac{\pi^2}{3}\left(\frac{k}{e}\right)^2 T$$

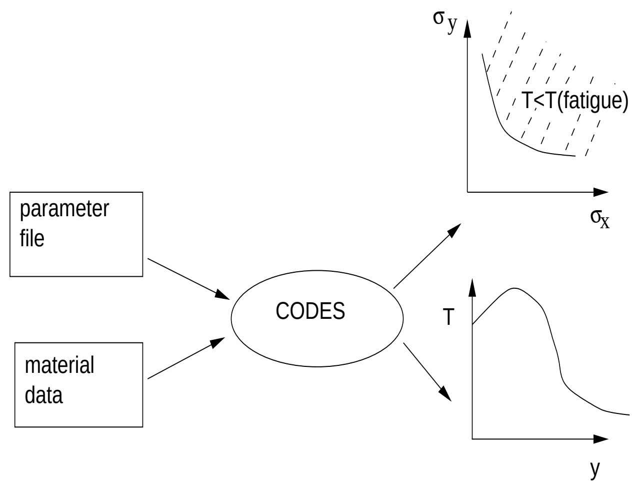
- we chose

$$K = K(\text{table,room}) \times \frac{K_{WF}(\frac{\text{room+fatigue}}{2})}{K_{WF}(\text{room})}$$

for copper : $K = 3.45\,10^2\ \text{Jm}^{-1}\text{K}^{-1}\text{s}^{-1}$
instead of $K = 3.93\,10^2\ \text{Jm}^{-1}\text{K}^{-1}\text{s}^{-1}$

Application to CLIC

# Codes

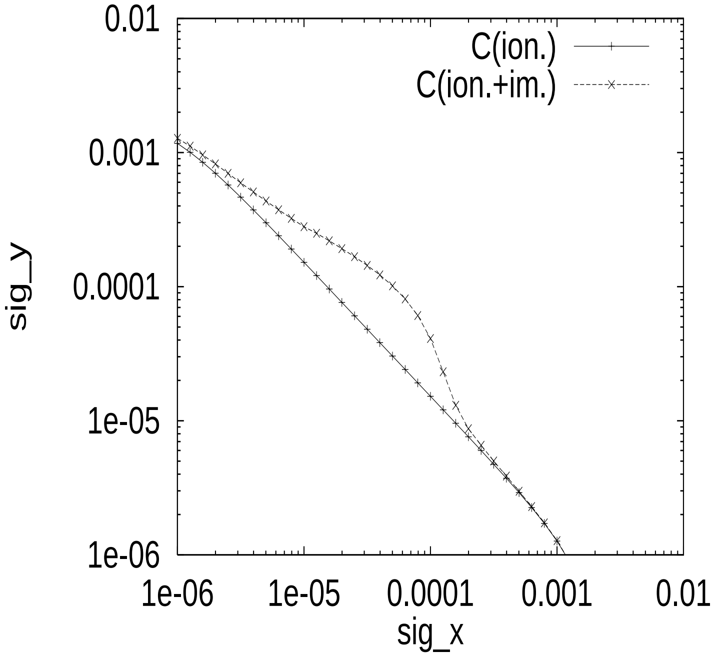


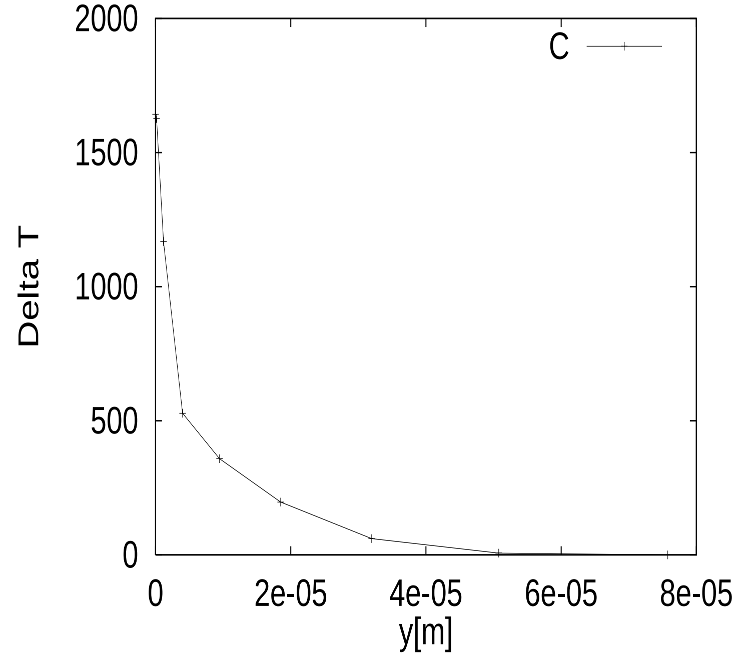

- Two codes in C (fast thanks to *Gauss-Legendre* integration)
- material data : $dE/dz, \rho, K, C_v, \sigma, T_{lim}$
- parameter file :

```
sig_z 30e-6                  sig_x 10e-6
n_part 4e9                   sig_y 50e-6
n_bunch 154                  a -25e-6
t_inter_bunch 666e-12        C
```

Application to CLIC

# Example : Output for carbon spoiler



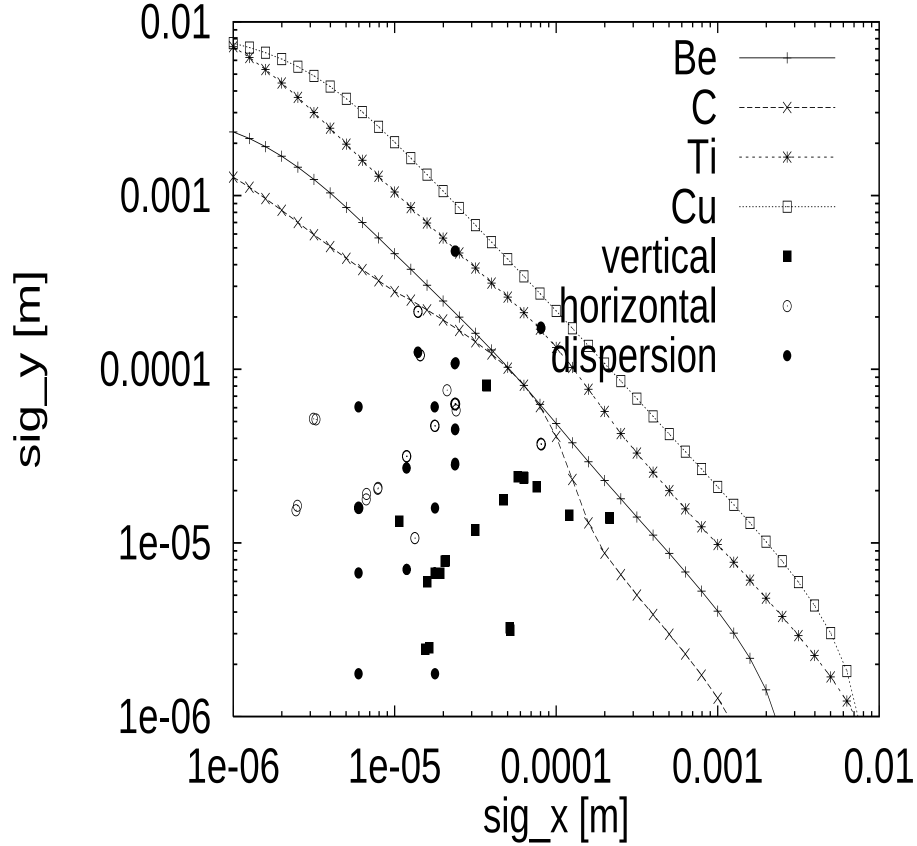




$\sigma_x = 125\,\mu m, \sigma_y = 23\,\mu m$

and $a = -21\,\mu m$

Application to CLIC

# Limit for several materials



## Conclusion

- 2D time dependant heat equation solved for ionisation and image current
- A code exists to specify the minimum beam sizes for any bunch train structure
- For CLIC : Spoilers in Be or C $\Rightarrow (\sigma_x \sigma_y) \approx (100\mu m)^2$